

MATHEMATICAL EXPERIENCE WITHIN CABRI-II MICROWORLD : CONSTRUCTION AND INTERPRETATION OF ALGEBRAIC EXPRESSIONS

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Abstract: We present results about the set up of scenarios on the learning of algebra within Cabri-II microworld. We started our teaching experiment in a french highschool with a tenth grade school class (16 years old), introducing the simulation in Cabri-II of machines to draw curves (see Hoyos et al. 1998). Here, we worked searching the establishment of connections between the geometrical properties and their corresponding algebraic symbolization. Afterwards, we inquired about the state of these connections by solving tasks of algebraical inequality. The results of the test that we designed supported one hypothesis of the present paper: in the execution of techniques and procedures based on the literal calculus one of the principal difficulties to overcome is the construction of means of control for the algebraical executions involved. Also, these results indicated a necessity of approach our work within Cabri-II to the graphical interpretation of algebraic expressions.

Introduction

In this paper we present results of a research project about the set up of scenarios on the learning of algebra within Cabri-II microworld. This research project was born after carrying out a wide theoretical observational study performed with students of approximately 17 years of age (who were studying the second year of high school in Mexico City); this study was inscribed in the inquiry about the construction of meaning of the lineal equations with two unknown values (see Hoyos 96, 97 and 98). Therein we sought to identify what the links that students established among the operational uses of algebraic variables were, which these take place in the tasks of translating from the geometric field to the algebraic one (analytical uses). Also we reported some of the difficulties which high school students present in the execution of algebraic tasks of syntactic manipulation.

In April 1997, we began our learning experiment in a French high school with a tenth grade school class (16 years olds). We started introducing the simulation in Cabri-II of machines to draw curves (see Hoyos et al. 1998). We chose Cabri-II microworld because some characteristics of this microworld incite pupils to the perception of geometrical invariant properties: "nous faisons l'hypothèse que l'ordinateur peut guider l'élève dans l'exploration de dessins en l'incitant à repérer des invariants entre plusieurs dessins, invariants pouvant déboucher sur la mise en évidence de notions et propriétés géométriques générales. En effet, ... il peut proposer plusieurs dessins représentant un

même ensemble de données théoriques." (See Bellemain 1992).

The activity in the scenario of simulation developed round the modeling of mechanisms of tracing of curves, the subsequent visualization of the curve(s) obtained and the task of finding the corresponding equation(s). Our hypothesis was that this would allow the pupil to establish connections between the geometrical properties and their corresponding algebraic symbolization.

Afterwards, we inquired about the state of these connections by solving tasks of algebraical inequality. The results in the application of the test that we designed (we present some of the items we included, below) indicated the existence of a very low level of success in the algebraical manipulations which, nevertheless, is usual in the school grade in question. These results supported one hypothesis of the present paper: in the execution of the techniques and procedures based on the literal calculus (as in the procedures which are carried out to solve an equation, or in the algebraical treatments to reach factorization or quotients of linear terms) one of the principal difficulties to overcome is the construction of means of control for the algebraical executions involved. Also, these results indicated a necessity of the approach our work within Cabri-II to the graphical interpretation of algebraic expressions.

Finally, in sense mentioned by Confrey (see 1993: "... technologies -that is, any significant tool- necessarily alter the character of knowledge."), probably we have evidences that our accomplishments within Cabri-II are significant tools for analytical uses of algebra.

Theoretical Underpinnings

The Computer: A Tool for the Construction of Meanings

One of the principal points of the theoretical framework sustaining this research is the conception of the construction of mathematical meaning within a differentiated support, or assistant to learner's system, of which the structure promotes the establishment of connections with other nomenclatures, with other mathematical domains. Within a system of this class (Noss and Hoyles 1996), the meaning's construction idea appeals to the presence of a structure over which the students may be able to construct and reconstruct, in the ways that they choose as the appropriate to their elaboration of the mathematical meanings in discussion effort's, with a support (real or virtual).

Indeed, these authors (Noss and Hoyles 1996) conceive the computer as integrating an educational support system in which elements are knowledge, student(s), teacher and the environment. They think of it as a system with several components that make it very advantageous: a) the fluid and flexible computational or informatical environment, that is under the control of the pupils; b) the possibility to indicate various trajectories to use instead of pointing to a unique and directed goal; c) there is a local support structure, to which the student has access and that is a product of his comprehension as well as a cultural product about it, built by others; d) there is a global support structure, that happens to be comprehended by the user, and that emerge of connections that become performed by him while he uses it.

Scenarios on the Learning of Algebra within Cabri-II Microworld

The microworld Cabri-II allows the integration of an educational support system of this kind because its characteristics of direct manipulation promote in students the perception of invariant geometrical properties (see Bellemain 1992).

In fact, the solid mathematical structure with which Cabri-II provides the student is a factor that promotes his advance in the construction of knowledge. Such a structure (see Noss and Hoyles 1996) is a determinant factor in the development of mathematical meaning: "It is the web of relationships and objets offered by the computer that can act both as a support for developing new meanings, as a means for transcending that support. For by manipulating objets and articulating the relationships between them, a dual action /notational framework is developed which can begin to be a new resource, one which is not so dependent on the medium for its expression." (Noss & Hoyles 1996)

Mathematical Experience and the Introduction of History's 'Voices' in the Classroom

Finally, another aspect that we have included for the design of the scenarios, it have been to introduce 'voices' of the history in the classroom -the Descartes'voice in this case, like else additional factor that would make possible to pupil the establishment of links between the voice and his/her own interpretations: "Each of these expressions conveys a content, an organization of the discourse and the cultural horizon of the historical leap. Referring to Bachtin (1968) and Wertsch (1991), we called these expressions 'voices'. Performing suitable task proposed by the teacher, the student may try to make connections between the voice and his/her own interpretations, conceptions, experiences and personal senses (Leontiev, 1978), and produce an echo, i.e. a link with the voice made explicit trough a discourse." (Boero, et al. 1997-98)

According to Boero et al., there exist cultural expressions that represent - from a deep communicative point of view - important leaps in the evolution of mathematics and science. It is through the execution of adequate tasks ("a game of voices and echoes") posed by the teacher, that the student may try to establish connections between this introduced 'voice' -the notational Cartesian geometrical-algebraic system, in this case, and his own interpretations. All the produced voices would contribute to produce an echo or link with the introduced voice.

Our Teaching Experiment in the 10th Grade Class

Simulation of Drawing Machines within Cabri-II

We began by introducing the simulation on Cabri-II of Descartes' machine, to plot hyperbolas. This machine is made of a mechanism, composed of a system (RSQ) of a fixed slope that slides vertically and of a shaft (AF) tied to this system. This shaft is also articulated to a fixed point (F) on a horizontal axis (see Descartes 1637). By introducing the simulation on Cabri-II of the mechanical construction of the plots, we wish to get close to the fundamental elements of Descartes'analysis situations in order to obtain algebraic equations. This teaching experiment was complementary to the normal mathematics course of the first year of high school (tenth grade class). We set it up by constituting three sessions of practical work of one hour each.

The first task's sequence was: 1) Modeling the machine; 2) Algebraic characterization of the straight line's families that step in the plot of Descartes' hyperbolas; 3) Obtaining the equations.

Our Test: Solving Tasks of Algebraical Inequality

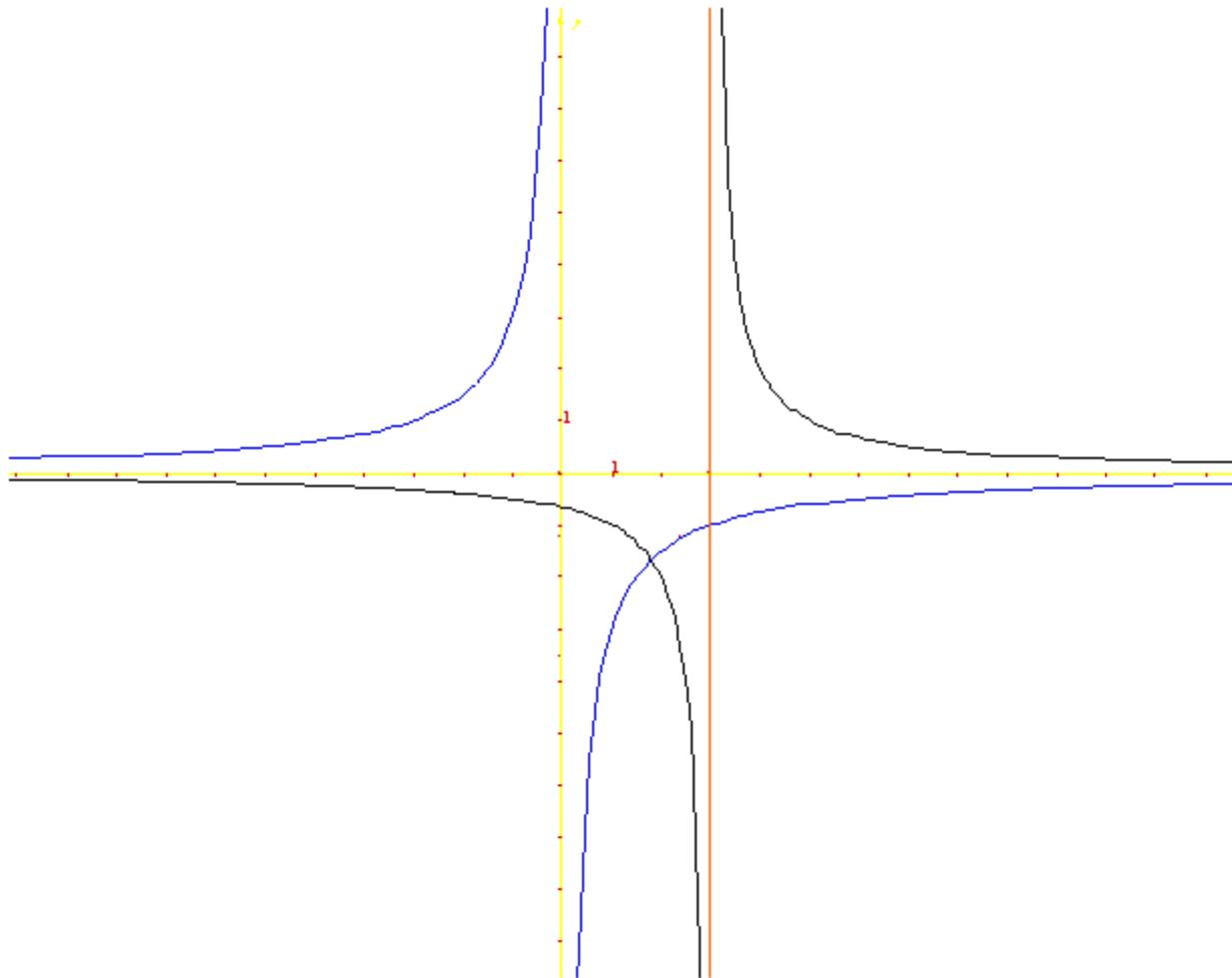
Following the scenario of simulation of Descartes' Machine, we applied a test. Some of the items in the test that we applied to our students were the following:

1. Résoudre les inéquations suivantes en donnant l'ensemble des solutions sous forme d'intervalle ou de réunion d'intervalles.

a) $(5x+2)(5-x) > 0$; b) $(x-2)^2 < (2x-3)^2$; 3) $1 < (x-1) < 5$

....

4. (Attention les axes sont identifiables par les graduations: le repère n'est pas normé). Sur ce graphique il y a deux représentations graphiques qui sont des hyperboles.



La première correspond à la fonction f définie par $f(x) = -\frac{3}{x}$,
la deuxième à la fonction g définie par $g(x) = \frac{2}{x-3}$

- Tracer en rouge la représentation graphique de f et en bleu celle de g .

- Résoudre graphiquement l'équation $f(x) = g(x)$ (encadrement(s) à 0,5 près)
 - Résoudre graphiquement l'inéquation $f(x) = g(x)$
5. En utilisant la calculatrice graphique, résoudre l'inéquation $x-2 = -2x+3$
Tracer la graphique obtenu sur votre feuille et marquer en couleur l'ensemble des solutions sous forme d'intervalle ou de réunion d'intervalles.

In our test, only 27% of our students succeeded in obtaining the algebraic inequalities presented in exercise 5. Also, the results concerning exercise 4 indicated that the students had reached the new curves they had studied with certain familiarity -the hyperbolas: the students correctly used their calculators to finger, identify and color the functional expressions given. However, the students did not succeed in discerning - based on graphic tools given by their calculators- what was happening with regard to the functional variation mentioned by members of algebraic inequality. From the results of the application of said test, we concluded in the necessity of expanding our teaching experiment with a practical work on the graphical representation of algebraic expressions.

Graphical Representation of Algebraic Expressions

The scenario of learning about the graphic representation of algebraic expressions which we elaborated, incorporates the use of old and new tools of the Cabri-II microworld: the dragging of any given point with the visualization of the change of its values; the use of a calculator of the proper microworld; the transfer of measurements; the tracing of curves through the track of the variable; and the geometric locus. Our hypothesis in accomplishing one scenario about this graphic representation was that the tools that the Cabri-II microworld provides for the graphical representation of functions permits the accomplishments of the cognitive actions which follow: dynamic perceptions of the elemental algebraical variables from different frames of representation -symbolical, graphical and numerical; recognition of the importance of the ortogonality on images-values of algebraic expressions -that we ask to pupils to denote with $f(x)$, in the Cartesian graphical representation of variables; dynamic perception of the print of variable point $P(x, f(x))$; dynamic perception of the functional dependence between x and $f(x)$.

The sequences about the graphical representation within Cabri-II were carried out during two extra-class practical work sessions, which amounted to two hours of directed work with the pupils. Only 7 volunteering pupils participated during the second. The two directed work sessions were completely videotaped and analyzed.

The Results: Mathematical Experience of the Pupils within Cabri-II Microworld

On the Modeling of Descartes'Machine

We asked pupils to simulate in Cabri-II the Descartes'machine. Indeed, the students (placed in Cabri-II) were committed in the proposed experiment. Several pupils were close to obtaining the equation of the curve. During the timed session, approximately 50 mins., despite its interest and the planned guidance, the students were only able to stablish the proportionality relations asked for. It was outside the class, later, that we

asked advanced pupils to finish that task. After reworking approximately ten minutes, they wrote the equation in question: $1/x-2+x = y$.

Graphical Treatment of Algebraical Inequalities within Cabri-II

Here we identified two principal actions that are not spontaneous for the student in his/her eagerness to obtain the graphic representation of a given algebraic expression: a) the dragging of the point M (the variable point $(x,0)$); and b) the simultaneous visualization in the screen of the movement of M and the effect that this provokes in the $f(x)$.

We believe that these actions are subject for to learn and when the pupils exert them, become fundamental elements in the understanding of the functional dependence of the values x and $f(x)$.

Also, this research work showed us the differences between the use of Cabri-II software and the usual graphical calculators: Cabri-II was revealed as a convenient microworld in the construction of means of control for the algebraic manipulations which the high school students usually carry out when solving algebraic inequalities. In other words, the graphication tools available in a usual graphical calculator provide us with finished graphical representations. Therefore, the students have difficulty in perceiving the variation of the values of the algebraic expressions, i.e., of the function; a perception that occupies an imminent role to succeed in the solution of the inequalities presented in our test.

Conclusions

It is probable that the set of actions executed by the students within Cabri-II has turned fundamental for the establishment of the connections sought, because the accomplishment of this practical work conducted the pupils to the revision of their own syntactical procedures of solution about algebraic inequalities; and it gave them a chance to validate their usual algebraic manipulations.

Also, we think that this working perspective for the learning of algebra contribute with a formulation of the problem of establishment of links between graphical and algebraical representations, like a problem of mathematical experience supported in the construction and interpretation of algebraical expressions by a dynamic draw of curves within Cabri-II, the description of their invariant geometrical properties, and the graphical representation of algebraical variables.